INTERNET APPENDIX
Dominant Currency Debt

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Abstract

This Internet Appendix is the companion document to the paper "Dominant Currency Debt" by Egemen Eren and Semyon Malamud published at the *Journal of Financial Economics*.

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A. Appendix to Section 2: Theory

Proposition A.1 *Suppose that firms have a possibility of hedging foreign exchange risk by acquiring $h_t \geq 0$ units of a financial derivative contract with a payoff of $X_{t+1} \geq 0$ and a price of $E_t[M_{t,t+1}X_{t+1}]$ to be paid at time t. The firm always chooses $h_t = 0$.*

The intuition behind this result is straightforward. Hedging effectively plays a role of investment, and the firm only gets the payoff X_{t+1} from this investment in good (survival) states, while paying the market price at time t to get the payoff in all states. Thus, hedging is just a transfer of funds from shareholders to debt–holders, and firms optimally decide to minimize this transfer.[1]

Proof of Proposition A.1. The maximization problem is

$$
\max_{h_t} \left\{ -E_t[M_{t,t+1}X_{t+1}]h_t \right.
$$
$$
\left. + E_t\left[M_{t,t+1}\int_{\Omega_{t+1}Z_{t+1}>\mathcal{B}_{t+1}(B_t)-h_t(1-\tau)X_{t+1}} (\Omega_{t+1}Z_{t+1} - \mathcal{B}_{t+1}(B_t) + h_t(1-\tau)X_{t+1})\phi(Z_{t+1})dZ_{t+1}\right] \right\}.
$$

The derivative of this objective function with respect to h_t is given by

$$
-E_t[M_{t,t+1}X_{t+1}] \;+\; (1-\tau)E_t\left[M_{t,t+1}X_{t+1}\left(1 - \Phi\left(\frac{\mathcal{B}_{t+1}(B_t) - h_t(1-\tau)X_{t+1}}{\Omega_{t+1}}\right)\right)\right] \;<\; 0\,,
$$

and hence $h_t = 0$ is optimal. Q.E.D.

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[1]There is ample evidence that firms often choose not to hedge their foreign currency risk. See, for example, Bodnár (2006) who shows that only 4% of Hungarian firms with foreign currency debt hedge their currency risk exposure. Furthermore, according to Salomao and Varela (forthcoming): "data from the Central Bank of Peru reveals that only 6% of firms borrowing in foreign currency employ financial instruments to hedge the exchange rate risk, and a similar number is found in Brazil." Du and Schreger (2017) also provide evidence that firms do not fully hedge their currency risk exposures. See also Niepmann and Schmidt-Eisenlohr (2019), Bruno and Shin (2017). While it is known that costly external financing makes hedging optimal (Froot, Scharfstein, and Stein, 1993; Hugonnier, Malamud, and Morellec, 2015), Rampini, Sufi, and Viswanathan (2014) show both theoretically and empirically that, in fact, more financially constrained firms hedge less.

Proof of Theorem 2.1. Firm's problem is to maximize

$$\sum_j E_t \left[M_{t,t+1} \left[\left(1 - (1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} \right) (1+c)\mathcal{E}_{j,i,t+1} \right] \right] B_{j,t}(1-q(j))$$

$$+ \; E_t \left[M_{t,t+1} \left[-\mathcal{B}_{t+1}(B_t)\left(1 - \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} \right) + \Omega_{t+1}\ell(\ell+1)^{-1}\left(1 - \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell+1} \right) \right] \right]$$

Differentiating, we get from the standard Kuhn–Tucker conditions that borrowing only in dollars is optimal if and only if

$$E_t \left[M_{t,t+1} \left[\left(1 - (1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} \right) (1+c)\mathcal{E}_{j,i,t+1} \right] \right] (1-q(j))$$

$$+ E_t \left[M_{t,t+1} \left[\left(-\ell(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell-1} \Omega_{t+1}^{-1} \right) (1+c)\mathcal{E}_{\$,i,t+1}(1+c(1-\tau))\mathcal{E}_{j,i,t+1} \right] \right] B_{\$,t}(1-q(\$))$$

$$- (1+c(1-\tau))E_t \left[M_{t,t+1}\mathcal{E}_{j,i,t+1} \right]$$

$$+ E_t \left[M_{t,t+1}(\ell+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} (1+c(1-\tau))\mathcal{E}_{j,i,t+1} \right.$$

$$\left. - \ell\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} (1+c(1-\tau))\mathcal{E}_{j,i,t+1} \right] \leq 0$$

for all j with the identity for $j = \$$. This inequality can be rewritten as

$$E_t[M_{t,t+1}\mathcal{E}_{j,i,t+1}]((1-q(j))(1+c) - (1+c(1-\tau)))$$

$$\leq \; E_t \left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} \mathcal{E}_{j,i,t+1} \right] ((1-\rho)(1+c)[(1-q(j)) + \ell(1-q(\$))] - (1+c(1-\tau)))$$

At the same time, for the dollar debt we get

$$E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]((1-q(\$))(1+c) - (1+c(1-\tau)))$$

$$= \; E_t \left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell} \mathcal{E}_{\$,i,t+1} \right] ((1+\ell)(1-\rho)(1+c)(1-q(\$)) - (1+c(1-\tau)))$$

implying that

$$B_{\$,t}(1+c(1-\tau)) \; = \; \left(\frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\mathcal{E}_{\$,i,t+1}\Omega_{t+1}^{-\ell} \right]} \right)^{\ell-1},$$

3

and we get the Kuhn–Tucker conditions

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} \frac{E_t[M_{t,t+1}\mathcal{E}_{j,i,t+1}]}{E_t\left[M_{t,t+1}\Omega_{t+1}^{-\ell}\mathcal{E}_{j,i,t+1}\right]} \leq \frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\mathcal{E}_{\$,i,t+1}\Omega_{t+1}^{-\ell}\right]},$$

and the claim follows. Q.E.D.

The next lemma formulates the optimality in terms of the pricing kernel $M_{t,t+1}^k = M_{t,t+1}^\$\mathcal{E}_{k,t,t+1}$ in a different currency k.

Lemma A.2 *Issuing in dollars is optimal if and only if*

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} \frac{E_t[M_{t,t+1}^k\mathcal{E}_{j,k,t+1}]}{E_t\left[M_{t,t+1}^k\tilde{\Omega}_{t+1}^{-\ell}\mathcal{E}_{j,k,t+1}\mathcal{E}_{\$,k,t+1}^\ell\right]} \leq \frac{E_t[M_{t,t+1}^k\mathcal{E}_{\$,k,t+1}]}{E_t\left[M_{t,t+1}^k\tilde{\Omega}_{t+1}^{-\ell}\mathcal{E}_{\$,k,t+1}^{1+\ell}\right]}$$

for all j.

Proof. The currency–k price of debt denominated in currency j satisfies

$$\delta^j(B_t,k) = E_t[M_{t,t+1}^k\left(1 - (1-\rho)\Phi(\Psi_{t+1}(B_t))\right)(1+c)\mathcal{E}_{j,t+1}/\mathcal{E}_{k,t+1}]$$

where $M_{t,t+1}^k = M_{t,t+1}^\$\mathcal{E}_{k,t,t+1}$ is the pricing kernel in currency k.

Let now $\tilde{V}_t = V_t/\mathcal{E}_{k,t}$ be the firm equity value in dollars and similarly $\tilde{\Omega} = \Omega/\mathcal{E}_k$ and $\tilde{\mathcal{B}}_{t+1} = \mathcal{B}_{t+1}/\mathcal{E}_{k,t+1}$ is the debt payoff denominated. Then,

$$\tilde{V}_t = V_t/\mathcal{E}_{k,t} = \tilde{\Omega}_t Z_t + \max_{B_t}\left\{\sum_{j=1}^N \delta^j(B_t,k)B_{j,t}(1-q(j)) + E_t[M_{t,t+1}^k\max\{\tilde{V}_{t+1} - \tilde{\mathcal{B}}_{t+1}(B_t),0\}]\right\}$$

and thus nothing changes. Thus, repeating the above argument, dollar debt is optimal if and only if

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} \frac{E_t[M_{t,t+1}^k\mathcal{E}_{j,k,t+1}]}{E_t\left[M_{t,t+1}^k\tilde{\Omega}_{t+1}^{-\ell}\mathcal{E}_{j,k,t+1}\mathcal{E}_{\$,k,t+1}^\ell\right]} \leq \frac{E_t[M_{t,t+1}^k\mathcal{E}_{\$,k,t+1}]}{E_t\left[M_{t,t+1}^k\tilde{\Omega}_{t+1}^{-\ell}\mathcal{E}_{\$,k,t+1}^{1+\ell}\right]}$$

Q.E.D.

Proof of Theorem 2.3. follows from the following known result.

Lemma A.3 *Suppose that f, g are monotone decreasing and bounded. Then,*

$$\text{Cov}_t(f(X), g(X)) \geq 0$$

for any bounded random variable X.

We need to compute

$$IRP_t = \frac{e^{r_t}\text{Cov}_t(M_{t,t+1}, \mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t+1}]}.$$

For simplicity, we will assume that all idiosyncratic shocks are identically zero. Define $\tilde{a}_t = -\log S_t$.

Our goal is to prove that

$$IRP_t + 1 = \frac{E_t[M_{t,t+1}\mathcal{P}_{i,t,t+1}]}{E_t[M_{t,t+1}]E_t[\mathcal{P}_{i,t,t+1}]}$$

$$= \frac{E_t[e^{(\phi+\gamma)\tilde{a}_{t+1}}]}{E_t[e^{\gamma\tilde{a}_{t+1}}]E_t[e^{\phi\tilde{a}_{t+1}}]}$$

is monotone increasing in ϕ. We have

$$\frac{\partial}{\partial\phi}\log(IRP_t(\phi) + 1) = \frac{E_t[e^{\tilde{a}_{t+1}(\phi+\gamma)}\tilde{a}_{t+1}]}{E_t[e^{\tilde{a}_{t+1}(\phi+\gamma)}]} - \frac{E_t[e^{\tilde{a}_{t+1}\phi}\tilde{a}_{t+1}]}{E_t[e^{\tilde{a}_{t+1}\phi}]}$$

Making a change of measure $d\tilde{P} = e^{\tilde{a}_{t+1}\phi}/E_t[e^{\tilde{a}_{t+1}\phi}]$, we can rewrite the required inequality as

$$\frac{\tilde{E}_t[e^{\gamma\tilde{a}_{t+1}}\tilde{a}_{t+1}]}{\tilde{E}_t[e^{\gamma\tilde{a}_{t+1}}]} > \tilde{E}_t[\tilde{a}_{t+1}],$$

which is a direct consequence of Lemma A.3. Q.E.D.

B. Appendix to Section 3: Evidence from Forward–Looking Measures

Table B.1. QRP, IRP, debt currency choice: Sample restricted to banks

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Banks	Banks	Banks	Banks	Banks	Banks
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP_{\text{€}/\$,t}^{2Y}$	–3.369***	–1.512*	–3.039***			
	(0.403)	(0.761)	(0.467)			
$IRP_{\$,t}^{2Y}$				–0.00366	0.000598	0.0487**
				(0.0204)	(0.0258)	(0.0211)
$IRP_{\text{€},t}^{2Y}$				–0.192***	–0.203***	–0.163***
				(0.0319)	(0.0513)	(0.0302)
Trend		X			X	
Control			X			X
Period	09q4–15q3	09q4–15q3	09q4–15q3	99q1–19q4	99q1–19q4	99q1–19q4
Observations	24	24	24	84	84	84
R–squared	0.644	0.708	0.710	0.284	0.285	0.478

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes only banks. Latest observed values of $QRP_{\text{€}/\$,t}^{2Y}$, $IRP_{\$,t}^{2Y}$ and $IRP_{\text{€},t}^{2Y}$ in a given quarter are used. $QRP_{\text{€}/\$,t}^{2Y}$ data come from Kremens and Martin (2019), and $IRP_{\$,t}^{2Y}$ and $IRP_{\text{€},t}^{2Y}$ come from Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.

Table B.2. QRP, IRP, debt currency choice: Sample restricted to non–banks

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Non–Banks	Non–Banks	Non–Banks	Non–Banks	Non–Banks	Non–Banks
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP_{€/\$,t}^{2Y}$	–2.750***	–1.759	–2.859***			
	(0.470)	(1.174)	(0.640)			
$IRP_{\$,t}^{2Y}$				–0.000896	–0.0124	0.0232
				(0.0207)	(0.0278)	(0.0252)
$IRP_{€,t}^{2Y}$				–0.196***	–0.166***	–0.203***
				(0.0253)	(0.0373)	(0.0226)
Trend		X			X	
Control			X			X
Period	09q4–15q3	09q4–15q3	09q4–15q3	99q1–19q4	99q1–19q4	99q1–19q4
Observations	24	24	24	84	84	84
R–squared	0.509	0.531	0.512	0.319	0.325	0.379

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes only non–banks. Latest observed values of $QRP_{€/\$,t}^{2Y}$, $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ in a given quarter are used. $QRP_{€/\$,t}^{2Y}$ data come from Kremens and Martin (2019), and $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ come from Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.
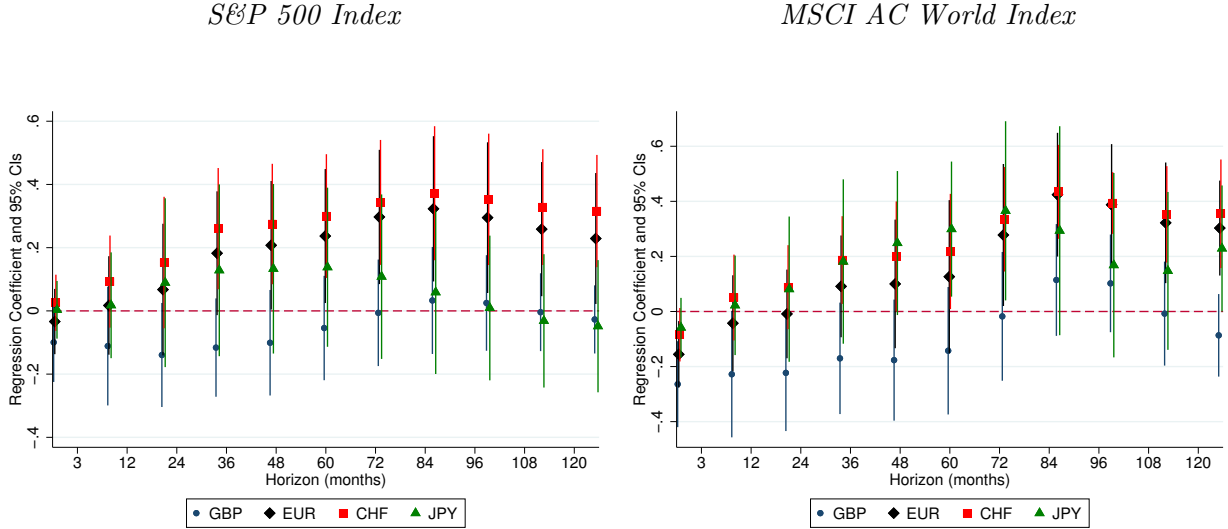
C. Appendix to Section 4: Evidence from Backward–Looking Measures

In order to complement the results from Section 4, in this appendix, we first redo the regressions in Section 4.1, using bilateral exchange rates as opposed to the dollar index. We show that the results are qualitatively similar, especially in terms of the patterns of short–term and longer–term covariances. Next, guided by these results, we compare the international debt share of the yen and the pound, currencies of countries that command a roughly similar share of the world economy. We show that the share of the pound and the yen in international debt markets behave in line with the risk properties of these currencies, in line with the mechanisms in our theory. We repeat the VAR analysis with the MSCI World Index instead of S&P 500 and show that our results are similar and even stronger. Finally, we report the results of the simple regressions between the dollar index and stock market indices with non–overlapping observations.

C.1 *Backward–looking results with bilateral exchange rates*

In this section, we provide the results for the same regressions as in Section 4.1, but using bilateral exchange rates for the dollar against four other major currencies. As Figure C.1 shows, the dominant currency condition (2) holds empirically with currency j being the euro (EUR), the yen (JPY), or the Swiss franc (CHF). The only exception is British pound (GBP), for which our empirical proxy estimates in Figure C.1 for the covariance in (2) have a negative sign. However, these covariance estimates are statistically insignificantly different from zero at the horizons of average debt maturity of firms. Thus, even absent differences in issuance costs, firms would strictly prefer issuing debt denominated in dollars, even if they could issue in EUR, JPY, or CHF. And even a slight difference in issuance costs favouring dollar to GBP would also make dollar immediately dominate over GBP.

Fig. C.1. The betas of the bilateral exchange rate of the dollar against major currencies with respect to stock market indices

S&P 500 Index   *MSCI AC World Index*
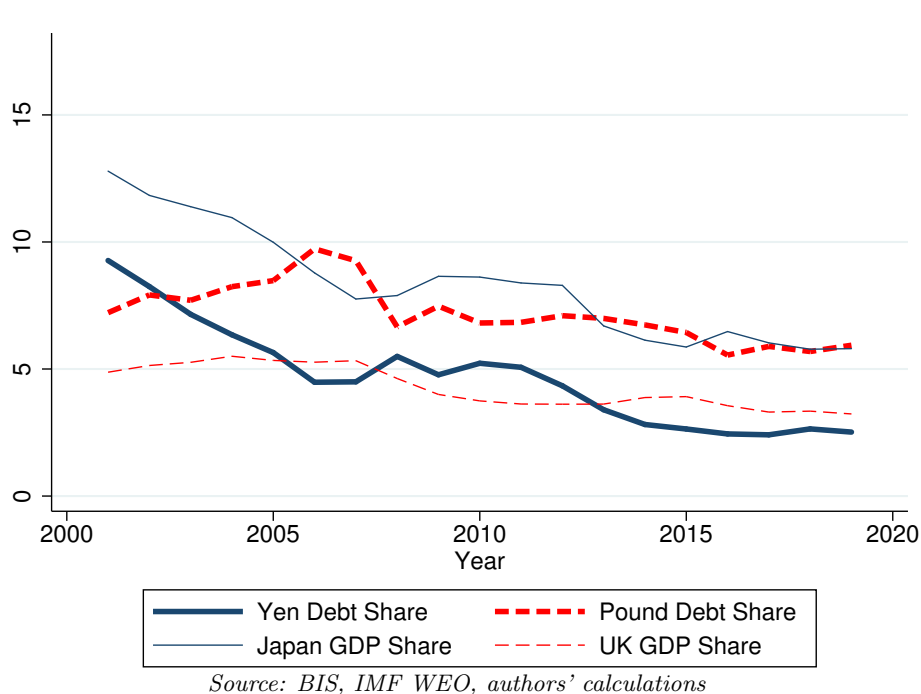
Notes: The graph on the left–hand side herein reports the regression coefficients β_h from the regressions (4) using the S&P 500 index. The graph on the right–hand side reports the regression coefficients from the regressions (4) using the MSCI AC World Index. The dots show the corresponding values of the β_h coefficients, while the lines show the 95% confidence intervals for these coefficients. Standard errors are corrected using the Newey–West procedure with the number of lags being equal to the horizon h of returns for each respective regression. The sample period for the S&P 500 goes from January 1973–December 2019. The sample period for the MSCI AC World Index goes from January 1988–December 2019 since data are only available starting from 1988.

C.2   *Yen vs. Pound*

As we show in Section C.1, the risk properties of the dollar alone can explain why the dollar dominates the euro, the yen and the Swiss franc in the sense of Theorem 2.1. One notable case is the pound: By Figure C.1, the pound has favorable risk properties for debt issuers compared to most of the other major currencies. In reality, there are many reasons why the pound may not be the most obvious competitor to the dollar, such as differences in the size of the economies, lower issuance costs for the dollar etc. However, it is reasonable to compare the dynamics of debt issuance in GBP to that in JPY, since Japan and the Great Britain have similar size in the world economy. In this case, Figure C.1 shows that (2) holds empirically if we replace $ with GBP and choose j=JPY. Hence, firms should strictly prefer issuing in GBP to issuing in JPY. Figure C.2 is in line with this prediction of our model. Indeed, surprisingly, despite the slightly larger share of

9

Japan in the world economy and lower nominal interest rates and inflation in Japan, the share of pound–denominated debt is higher than the share of yen–denominated debt.

Fig. C.2. The yen versus the pound

Source: BIS, IMF WEO, authors' calculations

C.3 *VAR Results using the MSCI World Index*

In this subsection, we redo the VAR analysis conducted in Section 4 with the MSCI All Country World Index instead of the S&P 500. Note that due to data availability, our sample period runs only from 1988 to 2019. We present the results of the estimation of the VAR(2) model as well the cumulative impulse response functions to a negative shock to the MSCI All Country World Index below.

C.4 *Simple Regressions Using Non–Overlapping Observations*

In this appendix, we repeat the exercise in Section 4.1, but with non–overlapping observations, for example we using data from January 1973, January 1978... etc to calculate five–year returns,
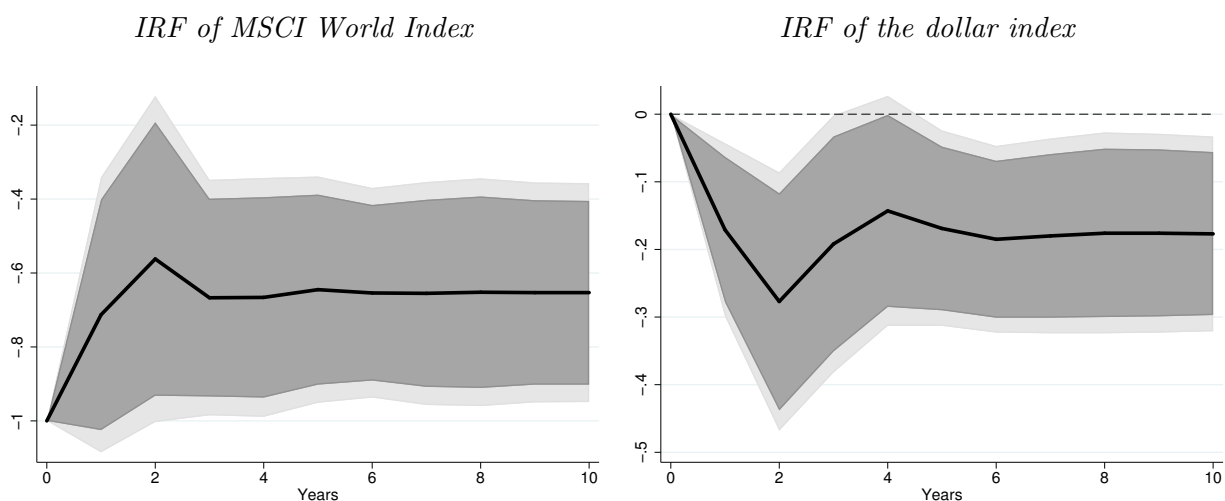
Table C.1. A VAR(2) model of the MSCI World Index and the FRED dollar index

	(1)	(2)
	$Ret\_MSCI_{t-1,t}$	$Ret\_USD_{t-1,t}$
$Ret\_MSCI_{t-2,t-1}$	−0.287	0.171**
	(0.208)	(0.0716)
$Ret\_MSCI_{t-3,t-2}$	−0.236	0.118
	(0.220)	(0.0756)
$Ret\_USD_{t-2,t-1}$	0.0140	0.219
	(0.536)	(0.184)
$Ret\_USD_{t-3,t-2}$	−0.0452	−0.286
	(0.519)	(0.179)
Observations	30	30
R–squared	0.1065	0.3433

Notes: Standard errors that are adjusted for small–sample degrees of freedom in parantheses. *, **, *** denote significance at the 10, 5, and 1% levels, respectively. The coefficients are from a VAR(2) model of the yearly returns on the MSCI World Index and the FRED dollar index against major currencies (DTWEXM) between 1988 and 2019. The variance–covariance matrix for the error terms is estimated as: $\hat{\Sigma} = \begin{pmatrix} 0.0270 & \\ --0.0027 & 0.0032 \end{pmatrix}$

and proceed similarly for other horizons, without any overlap between observations. Since our sample period is not sufficiently large, this approach necessarily leads to very small sample sizes. For example, since we have 46 years of data, five–year horizon only allows for 9 observations. We nevertheless report the results, for S&P 500 using a sample between 1973 and 2019, and for the MSCI World index, using a sample between 1988 and 2019, below. While in most cases, the results are not statistically significant, they follow a similar pattern that we have shown in Section 4.1.
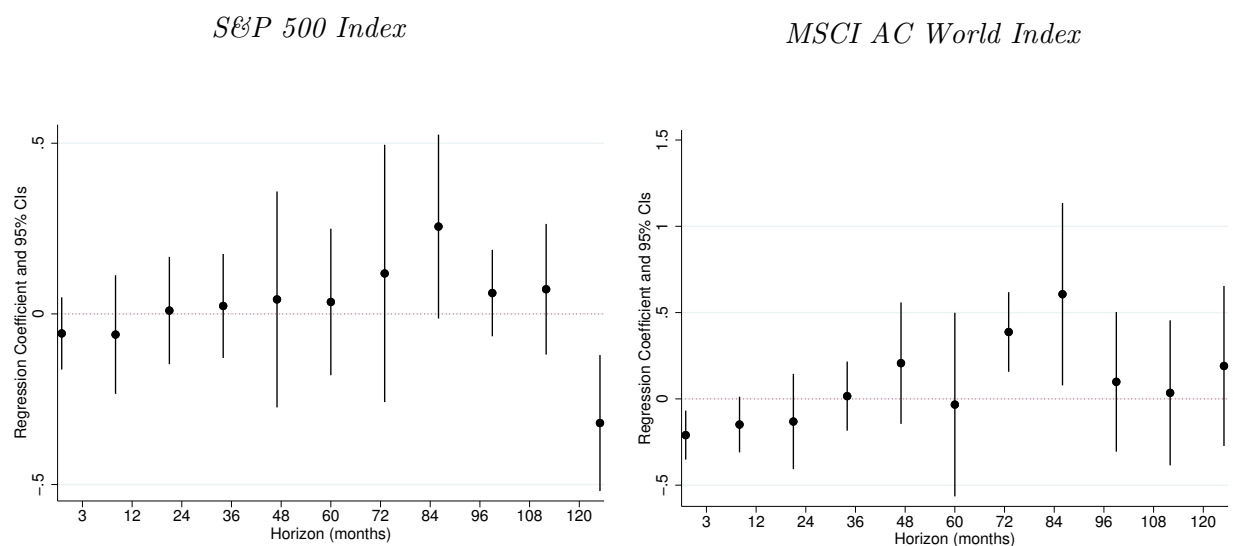
Fig. C.3. Cumulative Impulse Response Functions of a Shock to MSCI World Index

IRF of MSCI World Index

IRF of the dollar index

Notes: Figures show the cumulative impulse response functions of a negative 1 ppt shock to the MSCI World Index based on the estimates of a VAR(2) model of the yearly returns on the MSCI World Index and the FRED dollar index against major currencies (DTWEXM) between 1988 and 2019, reported in Table C.1. The lines in each graph represent the cumulative impulse response functions. The darker shaded areas represent the 90% confidence intervals, while the lighter shaded areas represent the 95% confidence intervals.

Fig. C.4. The betas of the USD index returns with respect to stock market indices using non–overlapping observations

S&P 500 Index

MSCI AC World Index

Notes: The graphs report the regression coefficients β_h from the regressions (4) using non–overlapping samples across different horizons, h. The round dots and the corresponding solid lines in both panels represent the point estimates, β_h, and 95% confidence intervals obtained using robust standard errors. The left–hand panel uses a sample period between January 1973 and December 2019. The right–hand panel uses a sample period between January 1988 and December 2019.

D. Appendix to Section 6: Dynamic Capital Structure

Firm's problem is to maximize

$$
\begin{aligned}
W_1(B_0) \;=\; & \sum_j E_1\left[M_{1,2}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,2}\right]B_{j,1}(1-q_c(j))\right] \\
+ \;& E_1\left[M_{1,2}\left[-\mathcal{B}_2(B_1+B_0)\left(1-\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\right)+\Omega_2\ell(\ell+1)^{-1}\left(1-\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell+1}\right)\right]\right]
\end{aligned}
$$

Differentiating, we get from the standard Kuhn–Tucker conditions that borrowing only in dollars is optimal if and only if

$$
\begin{aligned}
& E_1\left[M_{1,2}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,2}\right]\right](1-q_c) \\
& + E_1\left[M_{1,2}\left[\left(-\ell(1-\rho)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell-1}\Omega_2^{-1}\right)(1+c)(1+c(1-\tau))\mathcal{E}_{j,2}\right]\right]B_{\$,1}(1-q_c(\$)) \\
& - (1+c(1-\tau))E_1\left[M_{1,2}\mathcal{E}_{j,2}\right] \\
& + E_1\left[M_{1,2}(\ell+1)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,2}\right. \\
& \left. - \ell\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,2}\right] \le 0
\end{aligned}
$$

for all j with the identity for $j = \$$. This inequality can be rewritten as

$$
\begin{aligned}
& E_1[M_{1,2}\mathcal{E}_{j,2}]((1-q_c)(1+c)-(1+c(1-\tau))) \\
\le \;& E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\mathcal{E}_{j,2}\right]((1-\rho)(1+c)[(1-q_c)]-(1+c(1-\tau))) \\
+ \;& (1-q_c)\ell(1-\rho)(1+c)(1+c(1-\tau))E_1\left[M_{1,2}\left[\left(\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell-1}\Omega_2^{-1}\right)\mathcal{E}_{j,2}\right]\right]B_{\$,1}
\end{aligned}
$$

At the same time, for dollar debt we get

$$E_1[M_{1,2}]((1 - q_c(\$))(1 + c) - (1 + c(1 - \tau)))$$

$$= E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0) + (1 + c(1 - \tau))B_{1,\$}}{\Omega_2}\right)^\ell\right]((1 - \rho)(1 + c)[(1 - q)] - (1 + c(1 - \tau)))$$

$$+ (1 - q)\ell(1 - \rho)(1 + c)E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0) + (1 + c(1 - \tau))B_{1,\$}}{\Omega_2}\right)^\ell \frac{(1 + c(1 - \tau))B_{\$,1}}{\mathcal{B}_2(B_0) + (1 + c(1 - \tau))B_{1,\$}}\right]$$

and

$$E_1[M_{1,2}\mathcal{E}_{j,2}]((1 - q)(1 + c) - (1 + c(1 - \tau)))$$

$$\leq E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0) + (1 + c(1 - \tau))B_{1,\$}}{\Omega_2}\right)^\ell \mathcal{E}_{j,2}\right]((1 - \rho)(1 + c)[(1 - q)] - (1 + c(1 - \tau)))$$

$$+ (1 - q)\ell(1 - \rho)(1 + c)E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0) + (1 + c(1 - \tau))B_{1,\$}}{\Omega_2}\right)^\ell \frac{(1 + c(1 - \tau))B_{\$,1}}{\mathcal{B}_2(B_0) + (1 + c(1 - \tau))B_{1,\$}}\mathcal{E}_{j,2}\right]$$

Now, we will just be verifying the Kuhn–Tucker conditions at time zero when B_0 is purely in US dollars. In this case,

$$E_1[M_{1,2}\mathcal{E}_{j,2}]((1 - q)(1 + c) - (1 + c(1 - \tau)))$$

$$\leq E_1\left[M_{1,2}\left(\frac{(1 + c(1 - \tau))(B_{0,\$} + B_{1,\$})}{\Omega_2}\right)^\ell \mathcal{E}_{j,2}\right]((1 - \rho)(1 + c)[(1 - q)] - (1 + c(1 - \tau)))$$

$$+ (1 - q)\ell(1 - \rho)(1 + c)E_1\left[M_{1,2}\left(\frac{(1 + c(1 - \tau))(B_{0,\$} + B_{1,\$})}{\Omega_2}\right)^\ell \frac{B_{\$,1}}{B_{0,\$} + B_{1,\$}}\mathcal{E}_{j,2}\right]$$

while the dollar debt satisfies

$$E_1[M_{1,2}]((1 - q)(1 + c) - (1 + c(1 - \tau)))$$

$$= E_1\left[M_{1,2}\left(\frac{(1 + c(1 - \tau))(B_{0,\$} + B_{1,\$})}{\Omega_2}\right)^\ell\right]((1 - \rho)(1 + c)[(1 - q)] - (1 + c(1 - \tau))) \quad \text{(D.1)}$$

$$+ (1 - q)\ell(1 - \rho)(1 + c)E_1\left[M_{1,2}\left(\frac{(1 + c(1 - \tau))(B_{0,\$} + B_{1,\$})}{\Omega_2}\right)^\ell \frac{B_{\$,1}}{B_{0,\$} + B_{1,\$}}\right]$$

Dividing, we get

$$\frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]} = x(B_{0,\$} + B_{1,\$})^\ell + y(B_{0,\$} + B_{1,\$})^{\ell-1}B_{1,\$}$$

whereas

$$\frac{E_1[M_{1,2}\mathcal{E}_{j,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}]} \leq \frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]}$$

so the condition is still the same. Here, $x, y > 0$ (we assume that $x > 0$) are constants. This defines $B_{1,\$} = F(X_1, B_0)$. However, to derive the first order conditions, we will need to compute derivatives of F with respect to other debt components. To this end, we need to differentiate the implicit equation

$$\varepsilon_1 E_1[M_{1,2}]$$

$$= xE_1\left[M_{1,2}\left(\sum_j \mathcal{E}_{j,2}B_{j,0} + B_{1,\$}\right)^\ell \Omega_2^{-\ell}\right]$$

$$+ E_1\left[M_{1,2}\Omega_2^{-\ell}\left(\sum_j \mathcal{E}_{j,2}B_{j,0} + B_{1,\$}\right)^{\ell-1} B_{\$,1}^*\right]$$

Differentiating this identify with respect to $B_{j,0}^*$, we get

$$0 = x\ell E_1[M_{1,2}\Omega_2^{-\ell}(B_{0,\$} + B_{1,\$})^{\ell-1}(\mathcal{E}_{j,2} + B_{1,\$}')] + E_1\left[M_{1,2}\Omega_2^{-\ell}\left(B_{0,\$} + B_{1,\$}\right)^{\ell-1}\right] B_{\$,1}'$$

$$+ (\ell-1)E_1[M_{1,2}\Omega_2^{-\ell}(B_{0,\$} + B_{1,\$})^{\ell-2}(\mathcal{E}_{j,2} + B_{1,\$}')]B_{\$,1}$$

implying that

$$\frac{\partial B_{\$,1}}{\partial B_{j,0}} = -\frac{E_1[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}](x\ell(B_{0,\$} + B_{1,\$})^\ell + (\ell-1)(B_{0,\$} + B_{1,\$})^{\ell-1}B_{\$,1})}{E_1[M_{1,2}\Omega_2^{-\ell}](x\ell(B_{0,\$} + B_{1,\$})^\ell + [(\ell-1)B_{\$,1}(B_{0,\$} + B_{1,\$})^{\ell-1}] + (B_{0,\$} + B_{1,\$})^\ell)}$$

$$= -\frac{E_1[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]} \frac{\ell\varepsilon_1 E_1[M_{1,2}] - E_1[M_{1,2}\Omega_2^{-\ell}](B_{0,\$} + B_{1,\$})^{\ell-1}B_{\$,1}}{\ell\varepsilon_1 E_1[M_{1,2}] + E_1[M_{1,2}\Omega_2^{-\ell}](B_{0,\$} + B_{1,\$})^{\ell-1}B_{\$,0}}$$

Now we can solve the time zero problem. The objective with respect to B_0 is

$$\max_{B_0}\left\{\sum_j \delta_j(B_0)B_{j,0} + E[M_{0,1}\max\{W_1(B_0, X_1 Z_1) - \sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1}, 0\}]\right\}$$

16

where

$$\delta_{j,0}(B_0) \;=\; E[M_{0,1}(c_1 \mathcal{E}_{j,1} + \delta_{j,1}(B_0 + F(X_1, B_0)))\mathbf{1}_{W_1(B_0, X_1 Z_1) - \sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1} > 0}]$$

and where

$$\begin{aligned}
W_1(B_0, X_1) \;=\;& \delta_{\$,1}(B_0 + F(X_1, B_0)))F(X_1, B_0)(1 - q_c) \\
& + E_1[\mathbf{1}_{\Omega_2 > \mathcal{B}_2} M_{1,2}(\ell(\ell+1)^{-1}\Omega_2 - \mathcal{B}_2(B_1 + B_0))] \\
& + (\ell+1)^{-1} E_1\left[\mathbf{1}_{\Omega_2 > \mathcal{B}_2} M_{1,2}\Omega_2^{-\ell}(\mathcal{B}_2(B_1 + B_0))^{\ell+1}\right]
\end{aligned}$$

where we have defined

$$\delta_{j,1}(B_0 + F(X_1, B_0))) \;=\; E_1\left[M_{1,2}\mathbf{1}_{\Omega_2 > \mathcal{B}_2}\left[\left(1 - (1-\rho)\left(\frac{\mathcal{B}_2(F(X_1, B_0) + B_0)}{\Omega_2}\right)^\ell\right)(1 + c)\mathcal{E}_{j,2}\right]\right]$$

Now, by assumption, $Z_1 \sim \ell y^{\ell-1}$ on $[0,1]$. First, we need to figure out the threshold $\Theta_1(X_1, B_0)$ for default at time $t = 1$. It is determine via

$$W_1(B_0, X_1\Theta_1) - \sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1} \;=\; 0 \,.$$

Substituting, we get the following equation for Θ_1 evaluated as $B = B_{\$,0}$:

$$\begin{aligned}
& \delta_{\$,1}(B_0 + B_{\$,1}(\Theta_1), \Theta_1)B_{\$,1}(\Theta_1)(1 - q_c) \\
& + E_1[\mathbf{1}_{\Omega_2 > \mathcal{B}_2/\Theta_1} M_{1,2}(\ell(\ell+1)^{-1}\Omega_2\Theta_1 - (1 + c(1-\tau))(B_{\$,1}(\Theta_1) + B_0))] \\
& + (\ell+1)^{-1}(1 + c(1-\tau))^{\ell+1} E_1\left[M_{1,2}\mathbf{1}_{\Omega_2 > \mathcal{B}_2/\Theta_1}\Theta_1^{-\ell}\Omega_2^{-\ell}(B_{\$,1}(\Theta_1) + B_0)^{\ell+1}\right] \;=\; c_1 B_{\$,0}(1-\tau) \,.
\end{aligned}$$

We will impose a technical condition that at that threshold it is optimal not to issue any more debt (indeed, this is the default threshold, so it makes perfect sense). Hence, B_1 is not there at Θ_1 and now the equation for Θ_1 becomes much simpler. By assumption, there are always some states in which the firm survives at time $t = 2$. This is equivalent to $\min(\Omega_2)\Theta_1 > B_{\$,0}(1 + c(1-\tau))$. In this case,

$$\Theta_1 E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \;+\; B_{\$,0}^{\ell+1}\Theta_1^{-\ell}(\ell+1)^{-1} E_1[M_{1,2}\Omega_2^{-\ell}] \;=\; (1 + c(1-\tau) + c_1(1-\tau))B_{\$,0}$$

and hence

$$\Theta_1 = \Theta_1^* B_{\$,0}$$

where

$$\Theta_1^* E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] + (\Theta_1^*)^{-\ell}(\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}] = (1 + c(1-\tau) + c_1(1-\tau))$$

and this equation has a solution if and only if

$$\left(\frac{E_1[M_{1,2}\Omega_2]}{E_1[M_{1,2}\Omega_2^{-\ell}]}\right)^{-1/(\ell+1)} E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] + \left(\frac{E_1[M_{1,2}\Omega_2]}{E_1[M_{1,2}\Omega_2^{-\ell}]}\right)^{\ell/(\ell+1)} (\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}]$$

$$< (1 + c(1-\tau) + c_1(1-\tau))$$

That is, default threshold is homogeneous in $B_{\$,0}$ when no new debt is issued at the default threshold. Let us now verify the technical condition: It is equivalent to

$$\frac{1 + c(1-\tau)}{\min \Omega_2} E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] + \left(\frac{1 + c(1-\tau)}{\min \Omega_2}\right)^{-\ell}(\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}] > 1+c(1-\tau)+c_1(1-\tau)$$

Now, we can integrate the idiosyncratic shock away:

$$\delta_{j,0}(B_0) = E[M_{0,1}c_1\mathcal{E}_{j,1}(1 - \Theta_1(X_1,B_0)^\ell)] + E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \delta_{j,1}(B_0 + F(X_1q,B_0))\ell q^{\ell-1}dq]$$

$$= E[M_{0,1}c_1\mathcal{E}_{j,1}(1 - \Theta_1(X_1,B_0)^\ell)]$$

$$+ E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 E_1\left[M_{1,2}\left[\left(1 - (1-\rho)\left(\frac{(1+c(1-\tau))((F(X_1q,B_0)+B_0))}{\Omega_2 q}\right)^\ell\right)(1+c)\mathcal{E}_{j,2}\right]\right]\ell q^{\ell-1}dq]$$

$$= E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}(1+c)\mathcal{E}_{j,2})] - B_0^\ell E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^\ell]$$

$$- (1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\int_{B_0\Theta_1^*}^1 \mathcal{B}_2(q)^\ell q^{-1}dq]$$

We will also need

$$\bar{W}_1(B_0, X_1, Q) = \int_Q^1 W_1(B_0, X_1q)\ell q^{\ell-1}dq$$

18

and now we can finally write down the full value function:

$$\sum_j \delta_j(B_0)B_{j,0} - E[M_{0,1}\sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$

$$+ \ E[M_{0,1}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))]$$

Our objective is to verify the Kuhn–Tucker conditions at the pure dollar debt equilibrium. We thus need to verify (note that the terms with Θ_1' cancels out):

$$\delta_j(B_0)(1-q_c) \ + \ \frac{\partial}{\partial B_{j,0}}\delta_\$(B_0)B_{\$,0}(1-q_c) \ - \ E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$

$$+ \ E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))] < 0$$

for all $j \neq \$$. Now, we have

$$\frac{\partial}{\partial B_{j,0}}\delta_\$(B_0) \ = \ \frac{\partial}{\partial B_{j,0}}\bigg(E[M_{0,1}c_1(1-\Theta_1(X_1,B_0))^\ell]$$

$$+ \ E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \delta_{\$,1}(B_0+F(X_1 q,B_0))\ell q^{\ell-1}dq]\bigg)$$

$$= - \ \ell E[M_{0,1}c_1\Theta_1(X_1,B_0)^{\ell-1}\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)]$$

$$- \ \ell E[M_{0,1}\Theta_1(X_1,B_0)^{\ell-1}\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0))\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)]$$

$$+ E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \frac{\partial}{\partial B_{j,0}}\delta_{\$,1}(B_0+F(qX_1,B_0)))\ell q^{\ell-1}dq]$$

$$= -\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1+\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0),B_0))\Big)\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)\Big]$$

$$- \ E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^\ell$$

$$E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)(F(qX_1,B_0)+B_0))^{\ell-1}(1+c)\right]\ell q^{\ell-1}dq\Big]$$

$$= -\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1+\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0),B_0))\Big)\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)\Big]$$

$$- \ E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^\ell$$

$$E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)(F(qX_1,B_0)+B_0))^{\ell-1}(1+c)\right]\ell q^{\ell-1}dq\Big]$$

Furthermore, (recall that we are always evaluating the derivatives at $B_{k,0} = 0$ for all $k \neq \$$)

$$\frac{\partial}{\partial B_{j,0}}\delta_{\$,1}(B_0 + F(X_1, B_0))) = \frac{\partial}{\partial B_{j,0}}E_1\left[M_{1,2}\left[\left(1 - (1-\rho)\left(\frac{\mathcal{B}_2(F(X_1,B_0)+B_0)}{\Omega_2}\right)^\ell\right)(1+c)\right]\right]$$

$$= -\ell(1-\rho)(1+c(1-\tau))^\ell E_1\left[M_{1,2}\Omega_2^{-\ell}\left(\frac{\partial B_{\$,1}}{\partial B_{j,0}} + \mathcal{E}_{k,2}\right)(F(X_1,B_0)+B_0))^{\ell-1}(1+c)\right]$$

and

$$\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1, B_0) = \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{W_{1,X}(B_0, X_1\Theta_1)X_1}$$

whereas

$$\frac{\partial}{\partial B_{j,0}}W_1(B_0, X_1) = \frac{\partial}{\partial B_{j,0}}\left(\delta_{\$,1}(B_0 + B_{\$,1})B_{\$,1}(1 - q_c)\right.$$

$$+ E_1[M_{1,2}(\ell(\ell+1)^{-1}\Omega_2 - \mathcal{B}_2(B_1 + B_0))]$$

$$+ (\ell+1)^{-1}E_1\left[M_{1,2}\Omega_2^{-\ell}(\mathcal{B}_2(B_1+B_0))^{\ell+1}\right]\right)$$

$$= -\ell(1-\rho)(1+c(1-\tau))^\ell E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(1-q_c)$$

$$- (1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}] + (1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}+B_0)^\ell\mathcal{E}_{j,2}\right]$$

and

$$\frac{\partial}{\partial q}W_1(B_0, X_1 q) = \frac{\partial}{\partial q}\left(\delta_{\$,1}(B_0 + B_{\$,1}(q), q)B_{\$,1}(q)(1 - q_c)\right.$$

$$+ E_1[M_{1,2}(\ell(\ell+1)^{-1}\Omega_2 q - (1+c(1-\tau))(B_{\$,1}(q)+B_0))]$$

$$+ (\ell+1)^{-1}E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right]\right)$$

$$= \ell(1-\rho)(1+c(1-\tau))^\ell q^{-\ell-1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell(1+c)\right]B_{\$,1}(q)(1-q_c)$$

$$+ E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] - \ell(\ell+1)^{-1}(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}q^{-\ell-1}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right]$$

but at Θ_1 (since we assume that no new debt is issued at the default threshold)

$$
\begin{aligned}
\frac{\partial}{\partial q} W_1(B_0, X_1 q)|_{q=\Theta_1} &= E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \\
&\quad - \ell(\ell+1)^{-1}(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Theta_1^{-\ell-1}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right]
\end{aligned}
$$

Furthermore, we know

$$
E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] + B_{\$,0}^{\ell+1}\Theta_1^{-\ell-1}(\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}] = (1+c(1-\tau)+c_1(1-\tau))B_{\$,0}\Theta_1^{-1}
$$

and therefore we can rewrite

$$
\begin{aligned}
&E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] - \ell(\ell+1)^{-1}(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Theta_1^{-\ell-1}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right] \\
&= E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] - \ell\Big((1+c(1-\tau)+c_1(1-\tau))B_{\$,0}\Theta_1^{-1} - E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2]\Big) \\
&= \ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})
\end{aligned}
$$

and hence

$$
\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1, B_0) = \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}
$$

Note that, by the envelope condition, we just need to differentiate with respect to B_0, keeping $B_{1,\$}$ fixed.

Finally,

$$\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0, X_1, Q) = \int_Q^1 \frac{\partial}{\partial B_{j,0}}W_1(B_0, X_1 q)\ell q^{\ell-1}dq$$

$$= \int_Q^1 \ell\Bigg(-\ell(1-\rho)(1+c(1-\tau))^\ell q^{-\ell}E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c)$$

$$-(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}] + (1+c(1-\tau))^{\ell+1}q^{-\ell}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell \mathcal{E}_{j,2}\right]\Bigg)q^{\ell-1}dq$$

$$= -(1-Q^\ell)(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}]$$

$$+ \ell\int_Q^1 q^{-1}(1+c(1-\tau))^\ell\Bigg(-\ell(1-\rho)E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c)$$

$$+ (1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell \mathcal{E}_{j,2}\right]\Bigg)dq\,.$$

Thus,

$$\frac{\partial}{\partial B_{j,0}}\delta_\$(B_0)$$

$$= -\ell E\left[M_{0,1}\Theta_1^{\ell-1}\Big(c_1 + \delta_{\$,1}(B_0 + F(X_1\Theta_1(X_1, B_0), B_0))\Big)\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1, B_0)\right]$$

$$- E\Bigg[M_{0,1}\int_{\Theta_1(X_1, B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^\ell$$

$$E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}\Big(\frac{F(qX_1, B_0)}{\partial B_{j,0}} + \mathcal{E}_{j,2}\Big)(F(qX_1, B_0)+B_0))^{\ell-1}(1+c)\right]\ell q^{\ell-1}dq\Bigg]$$

$$= -\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1 + \delta_{\$,1}(B_0 + F(X_1\Theta_1(X_1, B_0), B_0))\Big)$$

$$\times \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}\Big]$$

$$- E\Bigg[M_{0,1}\int_{\Theta_1(X_1, B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^\ell$$

$$E_1\left[M_{1,2}\Omega_2^{-\ell}\Big(\frac{F(qX_1, B_0)}{\partial B_{j,0}} + \mathcal{E}_{j,2}\Big)(F(qX_1, B_0)+B_0))^{\ell-1}(1+c)\right]\ell q^{-1}dq\Bigg]$$

whereas

$$E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0, X_1, \Theta_1(X_1, B_0))]$$

$$= E\Bigg[M_{0,1}\Bigg(-(1-\Theta_1^\ell)(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}]$$

$$+ \ell\int_{\Theta_1}^1 q^{-1}(1+c(1-\tau))^\ell\Bigg(-\ell(1-\rho)E_1\Big[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\Big]B_{\$,1}(q)(1-q_c)$$

$$+ (1+c(1-\tau))^{\ell+1}E_1\Big[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell\mathcal{E}_{j,2}\Big]\Bigg)dq\Bigg)\Bigg]$$

To proceed further, we need to derive the first order approximation to the policy function $B_{\$,1}$ using (D.1) under the assumption that $((1-q)(1+c)-(1+c(1-\tau)))$ is small. In this case,

$$E_1[M_{1,2}]((1-q)(1+c)-(1+c(1-\tau)))$$

$$= E_1\Bigg[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^\ell\Bigg]((1-\rho)(1+c)[(1-q)]-(1+c(1-\tau)))$$

$$+ (1-q)\ell(1-\rho)(1+c)E_1\Bigg[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^\ell\frac{B_{\$,1}}{B_{0,\$}+B_{1,\$}}\Bigg]$$

Let

$$\varepsilon_1 = \frac{(1-q)(1+c)-(1+c(1-\tau))}{(1-q)\ell(1-\rho)(1+c)(1+c(1-\tau))^\ell}, \quad \varepsilon_2 = \frac{(1-q)(1-\rho)(1+c)-(1+c(1-\tau))}{(1-q)\ell(1-\rho)(1+c)(1+c(1-\tau))^\ell}$$

and assume they are small (this is indeed the case when τ is small). Then, the gains from debt issuance are small and hence debt is small. Thus, B_0 and B_1 are proportional to $\varepsilon_1^{1/\ell}$ and $B_{1,\$}(q)$ (when Ω_2 is multiplied by q) solves

$$q^\ell\frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]}\varepsilon_1 = (B_0+B_1)^{\ell-1}B_1 + x(B_0+B_1)^\ell$$

We will rescale them and denote $B_t^* = B_t/\varepsilon_1^{1/\ell}$. Then,

$$q^\ell\frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]} = (B_0^*+B_1^*)^{\ell-1}B_1^* + x(B_0^*+B_1^*)^\ell$$

23

where we assume that x is constant.

Now we can gather all the terms and rewrite the Kuhn–Tucker condition as

$$\delta_j(B_0)(1-q_c) \ + \ \frac{\partial}{\partial B_{j,0}}\delta_\$(B_0)B_{\$,0}(1-q_c) \ - \ E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$

$$+ \ E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))]$$

$$= \ \left(E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})] - B_0^\ell E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^\ell] \right.$$

$$\left. - (1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\int_{B_0\Theta_1^*}^1 \mathcal{B}_2(q)^\ell q^{-1}dq] \right)(1-q_c)$$

$$+ B_{\$,0}\left(-\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1 + \delta_{\$,1}(B_0 + F(X_1\Theta_1(X_1,B_0),B_0))\Big) \right.$$

$$\times \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2]-(1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}\Big]$$

$$- E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^\ell$$

$$E_1\left[M_{1,2}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)(F(qX_1,B_0)+B_0))^{\ell-1}(1+c)\right]\ell q^{-1}dq\Big] \right)(1-q_c)$$

$$- E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$

$$+ E\Big[M_{0,1}\Big(-(1-\Theta_1^\ell)(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}]$$

$$+ \ell\int_{\Theta_1}^1 q^{-1}(1+c(1-\tau))^\ell\Big(-\ell(1-\rho)E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c)$$

$$+ (1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell\mathcal{E}_{j,2}\right]\Big)dq\Big)\Big]$$

Now, since B_0 is small, we will only keep the highest order terms (those of order $B_{\$,0}^\ell$) and ignore

the terms of the order $o(B_{\$,0}^\ell)$. This gives

$$\delta_j(B_0)(1-q_c) + \frac{\partial}{\partial B_{j,0}}\delta_\$(B_0)B_{\$,0}(1-q_c) - E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$

$$+ E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))]$$

$$= \Bigg(E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}(1+c)\mathcal{E}_{j,2})] - B_0^\ell E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^\ell]$$

$$- (1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\int_{\Theta_1}^1 \mathcal{B}_2(q)^\ell q^{-1}dq]\Bigg)(1-q_c)$$

$$+ B_{\$,0}\Bigg(-\ell B_{\$,0}^{\ell-1}E\Big[M_{0,1}(\Theta_1^*)^{\ell-1}\Big(c_1 + E_1[M_{1,2}(1+c)]\Big)$$

$$\times \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}\Big]$$

$$- E\Big[M_{0,1}\int_{\Theta_1}^1 \ell(1-\rho)(1+c(1-\tau))^\ell$$

$$E_1\Big[M_{1,2}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}} + \mathcal{E}_{j,2}\Big)(F(qX_1,B_0)+B_0))^{\ell-1}(1+c)\Big]\ell q^{-1}dq\Big]\Bigg)(1-q_c)$$

$$- E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-B_{\$,0}^\ell(\Theta_1^*)^\ell)]$$

$$+ \Bigg(E\Big[M_{0,1}\Big(-(1-B_{\$,0}^\ell(\Theta_1^*)^\ell)(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}]$$

$$+ \ell\int_{\Theta_1}^1 q^{-1}(1+c(1-\tau))^\ell\Big(-\ell(1-\rho)E_1\Big[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\Big]B_{\$,1}(q)(1-q_c)$$

$$+ (1+c(1-\tau))^{\ell+1}E_1\Big[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell\mathcal{E}_{j,2}\Big]\Big)dq\Big)\Big]\Bigg) + o(\varepsilon_1^{1/\ell})$$

We can now regroup this as

$$E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}c\mathcal{E}_{j,2})](\tau - q_c)$$

$$+ B_{\$,0}^{\ell}\Bigg(- E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}](1 - q_c)$$

$$- \ell E\Bigg[M_{0,1}(\Theta_1^*)^{\ell-1}\Big(c_1 + E_1[M_{1,2}(1+c)]\Big) \times \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2] - (1 + c(1-\tau) + c_1(1-\tau))(\Theta_1^*)^{-1})}\Bigg](1 - q_c)$$

$$+ E[(M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1} + (1 + c(1-\tau))M_{0,2}\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}]\Bigg)$$

$$+ \int_{\Theta_1}^{1} q^{-1}$$

$$\Bigg(- (1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\mathcal{B}_2(q)^{\ell}(1 - q_c)$$

$$- \ell^2 E[M_{0,2}(1-\rho)\Omega_2^{-\ell}(1 + c(1-\tau))\Big(\frac{F(qX_1, B_0)}{\partial B_{j,0}} + \mathcal{E}_{j,2}\Big)\mathcal{B}_2(q)^{\ell-1}B_{\$,0}(1+c)(1 - q_c)$$

$$+ \ell E\Bigg[M_{0,2}(1 + c(1-\tau))^{\ell}\Big(- \ell(1-\rho)\Big[\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q) + B_0)^{\ell-1}(1+c)\Big]B_{\$,1}(q)(1 - q_c)$$

$$+ (1 + c(1-\tau))^{\ell+1}\Big[\Omega_2^{-\ell}(B_{\$,1}(q) + B_0)^{\ell}\mathcal{E}_{j,2}\Big]\Big)\Bigg)dq \ + \ o(\varepsilon_1^{1/\ell})$$

Recalling that we assume that q, τ, ρ are all of the order as ε_1. Thus,

$$-(1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\mathcal{B}_2(q)^{\ell}(1-q_c) + (1 + c(1-\tau))^{\ell+1}\Big[\Omega_2^{-\ell}(B_{\$,1}(q) + B_0)^{\ell}\mathcal{E}_{j,2}\Big] \ = \ O(\varepsilon_1^2)$$

and

$$B_{\$,0}^{\ell}(-E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}](1-q_c) + E[(M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1} + (1 + c(1-\tau))M_{0,2}\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}]) = O(\varepsilon_1^2).$$

Thus, we are only left with

$$E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}c\mathcal{E}_{j,2})](\tau - q_c)$$

$$+ B_{\$,0}^{\ell}\left(-\ell E\left[M_{0,1}(\Theta_1^*)^{\ell-1}\frac{c_1(1-\tau)\mathcal{E}_{j,1}\left(c_1 + E_1[M_{1,2}(1+c)]\right)}{\ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau) + c_1(1-\tau))(\Theta_1^*)^{-1})}\right](1-q_c)\right)$$

$$+ \int_{\Theta_1}^1 q^{-1}$$

$$\left(-\ell^2 E[M_{0,2}(1-\rho)\Omega_2^{-\ell}(1+c(1-\tau))\frac{F(qX_1,B_0)}{\partial B_{j,0}}\mathcal{B}_2(q)^{\ell-1}B_{\$,0}(1+c)(1-q_c)]\right.$$

$$\left.- \ell^2 E\left[M_{0,2}(1+c(1-\tau))^{\ell}\left((1-\rho)\left[\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q) + B_0)^{\ell}(1+c)\right](1-q_c)]\right)\right]\right)dq + o(\varepsilon_1^{1/\ell})$$

Now, the really surprising effect is that when $B_{\$,0}$ is close to zero, the integral produces a logarithm term through the following lemma.

Lemma D.1

$$\int_{\Theta_1^* B}^1 f(q)q^{-1}dq \approx -\log(\Theta_1^* B)f(\Theta_1^* B)$$

as $B \to 0$.

Thus, the key term is

$$E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}c\mathcal{E}_{j,2})](\tau - q_c(j)) + \ell^2(B_{\$,0})^{\ell}\log(B_{\$,0}\Theta_1^*)$$

$$E\left[M_{0,2}(1+c(1-\tau))^{\ell}(1-\rho)\left[\Omega_2^{-\ell}\mathcal{E}_{j,2}(1+c)\right](1-q_c(\$))\right]$$

which should be non–positive for $j \neq \$$ and zero for $j = \$$. This leads to the inequality

$$\frac{q_c(\$) - q_c(j)}{\tau - q_c(\$)} \leq \frac{E^{\$}[\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E^{\$}[\Omega_2^{-\ell}]}\frac{c_1 e^{-r_1(\$)} + c_2 e^{-r_2(\$)}}{c_1 e^{-r_1(j)} + c_2 e^{-r_2(j)}} - 1$$

$$= \frac{E^{\$}[\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]}\frac{c_1 e^{r_2(\$)-r_1(\$)} + c_2}{c_1 e^{r_2(j)-r_1(j)} + c_2} - 1$$

$$= \frac{\mathrm{Cov}^{\$}(\Omega_2^{-\ell}, \mathcal{E}_{j,2})}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]} + \frac{E^{\$}[\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]}\frac{c_1\left(e^{r_2(\$)-r_1(\$)} - e^{r_2(j)-r_1(j)}\right)}{c_1 e^{r_2(j)-r_1(j)} + c_2}$$

where we have used that $E^{\$}[\mathcal{E}_{j,2}] = e^{r_2(\$)-r_2(j)}$.

Proof of Proposition 6.1. We need to show that

$$-\sum_j E_1 \left[M_{1,2} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell \right) (1+c)\mathcal{E}_{j,2} \right] \right] B_{j,1}$$

$$+ E_1 \left[M_{1,2} \left[-\mathcal{B}_2(-B_1 + B_0) \left(1 - \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell \right) + \Omega_2 \ell (\ell+1)^{-1} \left(1 - \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell+1} \right) \right] \right]$$

is monotone decreasing in $B_{j,1}$ for $B_{j,1} \geq 0$. Taking the derivative, we get

$$- E_1 \left[M_{1,2} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell \right) (1+c)\mathcal{E}_{j,2} \right] \right]$$

$$- \sum_k E_1 \left[M_{1,2} \left[\left(\ell(1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell-1} \Omega_2^{-1} \right) (1+c)(1+c(1-\tau))\mathcal{E}_{j,2}\mathcal{E}_{k,2} \right] \right] B_{k,1}$$

$$+ (1 + c(1-\tau))E_1 \left[M_{1,2}\mathcal{E}_{j,2} \right]$$

$$- E_1 \left[M_{1,2}(\ell+1) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell (1+c(1-\tau))\mathcal{E}_{j,2} \right.$$

$$\left. + \ell \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell (1+c(1-\tau))\mathcal{E}_{j,2} \right]$$

$$= -E_1 \left[M_{1,2} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell \right) (1+c)\mathcal{E}_{j,2} \right] \right]$$

$$- \sum_k E_1 \left[M_{1,2} \left[\left(\ell(1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell-1} \Omega_2^{-1} \right) (1+c)(1+c(1-\tau))\mathcal{E}_{j,2}\mathcal{E}_{k,2} \right] \right] B_{k,1}$$

$$+ (1 + c(1-\tau))E_1 \left[M_{1,2}\mathcal{E}_{j,2} \right]$$

$$- E_1 \left[M_{1,2} \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell (1+c(1-\tau))\mathcal{E}_{j,2} \right]$$

$$= -c\tau E_1[M_{1,2}\mathcal{E}_{j,2}]$$

$$+ ((1-\rho)(1+c) - (1+c(1-\tau)))E_1 \left[M_{1,2} \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell \mathcal{E}_{j,2} \right]$$

$$- \sum_k E_1 \left[M_{1,2} \left[\left(\ell(1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell-1} \Omega_2^{-1} \right) (1+c)(1+c(1-\tau))\mathcal{E}_{j,2}\mathcal{E}_{k,2} \right] \right] B_{k,1}$$

$$\leq -c\tau E_1[M_{1,2}\mathcal{E}_{j,2}]$$

$$+ ((1-\rho)(1+c) - (1+c(1-\tau)))E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^\ell \mathcal{E}_{j,2}\right]$$

$$- \sum_k E_1\left[M_{1,2}\left[\left(\ell(1-\rho)\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^{\ell-1}\Omega_2^{-1}\right)(1+c)(1+c(1-\tau))\mathcal{E}_{j,2}\mathcal{E}_{k,2}\right]\right]B_{k,1}\mathbf{1}_{B_{k,1}\leq 0}$$

$$\leq -c\tau E_1[M_{1,2}\mathcal{E}_{j,2}]$$

$$+ ((1-\rho)(1+c) - (1+c(1-\tau)))E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^\ell \mathcal{E}_{j,2}\right]$$

$$+ \ell(1-\rho)(1+c)E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^\ell \mathcal{E}_{j,2}\right]$$

$$= -c\tau E_1[M_{1,2}\mathcal{E}_{j,2}] + ((1+\ell)(1-\rho)(1+c) - (1+c(1-\tau)))E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^\ell \mathcal{E}_{j,2}\right]$$

and hence the result holds if

$$\sup\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^\ell \leq \frac{c\tau}{(1+\ell)(1-\rho)(1+c) - (1+c(1-\tau))}.$$

Consider now the possibility that new debt is issued at time $t = 1$. The optimal amount of new debt in currency p satisfies

$$0 = -c\tau E_1[M_{1,2}\mathcal{E}_{p,2}]$$

$$+ ((1-\rho)(1+c) - (1+c(1-\tau)))E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^\ell \mathcal{E}_{p,2}\right]$$

$$- \sum_k E_1\left[M_{1,2}\left[\left(\ell(1-\rho)\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^{\ell-1}\Omega_2^{-1}\right)(1+c)(1+c(1-\tau))\mathcal{E}_{p,2}\mathcal{E}_{k,2}\right]\right]B_{k,1}$$

Define the matrix

$$\mathcal{P}_{j,k} = \left[\left(\ell(1-\rho)\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^{\ell-1}\Omega_2^{-1}\right)(1+c)(1+c(1-\tau))\mathcal{E}_{j,2}\mathcal{E}_{k,2}\right]$$

and the vector

$$Q_k = (((1-\rho)(1+c) - (1+c(1-\tau)))) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^\ell - c\tau)\mathcal{E}_{k,2} > 0.$$

Then, first order conditions can be rewritten as a system

$$E_1[M_{1,2}\mathcal{P}]B = E_1[M_{1,2}Q]$$

In the extreme case when all exchange rates are identical, it follows immediately that the total net acquired-back det $\sum_k B_k$ has to be strictly negative (meaning that the firm is issuing debt and is indifferent about re-shuffling because all currencies are identical). But one currencies are slightly different, the optimal debt issuance will feature approximately equal issuance amounts, all positive. Q.E.D.

E. Additional results: Local currency and dollar debt

The main goal of our paper is to explain the dominance of the dollar compared to other major international currencies primarily with global firms in mind. While it is not our primary focus to explain why firms in emerging markets issue debt in dollars as opposed to local currency, the mechanisms underlined in our paper do yield some predictions about that as well. In this appendix, we take as given the dominance of dollar among the major global currencies, and we investigate whether debt view can be used to explain the mixture of dollar– and local–currency denominated debt for non–financial firms in a cross–section of emerging market economies.

E.1 *Results*

We develop and test the predictions of an extension of our model using a cross–section of the emerging market economies for which data on corporate debt in different currencies are available.[2] We prove the following extension of Theorem 2.1 for the case wherein firms issue a mixture of local currency (LC) and dollar–denominated debt (see Theorem E.2 in the Appendix for the proof. Note that, while Proposition E.1 is a partial equilibrium result, it still holds true in general equilibrium when debt overhang costs are sufficiently small).

Proposition E.1 *Suppose that (1) $q = q(\$)$ (that is, issuing in LC costs the same as issuing in dollars); (2) the variance of all shocks is sufficiently small; and (3) issuing debt in both LC and dollars is optimal; (4) ℓ is close to 1. Then,*

(a) *the fraction $\frac{B_t}{B_t(\$)\mathcal{E}_{\$,i,t}}$ is monotone increasing in the covariance $\text{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ if and only if $B_t \geq B_t(\$)\mathcal{E}_{\$,i,t}$;*

(b) *the fraction $\frac{B_t}{B_t(\$)\mathcal{E}_{\$,i,t}}$ is always monotone decreasing in $\sigma_{i,\varepsilon}$.*

The intuition for the first theoretical result is that local currency debt partly replicates insurance properties of the dominant currency in downturns, while it is a better hedge against domestic

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productivity shocks. The second result is that volatile inflation generates volatility of profits which the firms avoid by issuing less local currency debt.

Items (a)–(b) of Proposition E.1 directly translate into the testable empirical hypotheses. We test the two implications of our theory:

1. The local currency share of corporate debt is *higher* for countries in which domestic inflation correlates more with US inflation when controlling for relevant factors.

2. Firms in countries with more volatile domestic inflation tend to have less debt denominated in local currency.

Fig. E.1. Mean of the local currency to USD debt ratio by country
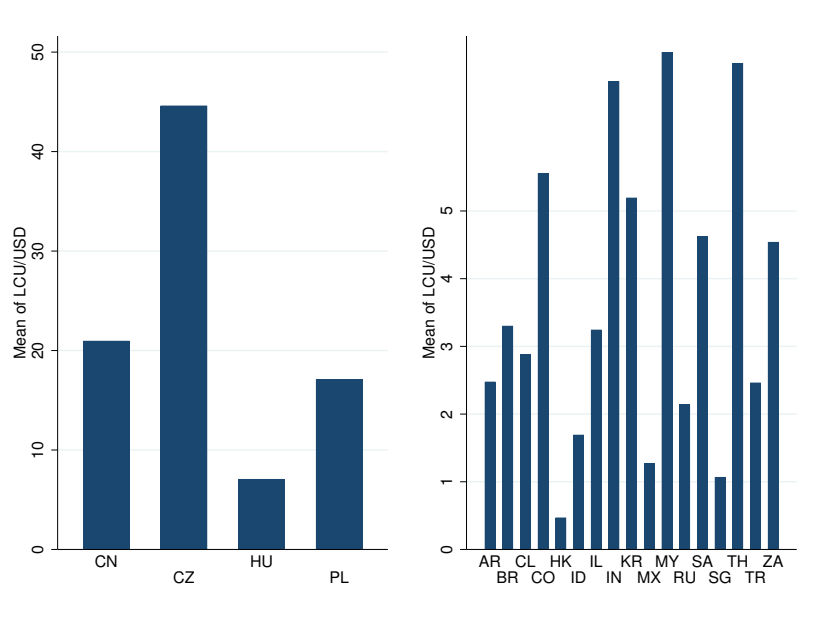
Source: IIF, authors' calculations

Figure E.1 shows the mean of the debt ratio, $\frac{\bar{LCU}}{USD}_i$, for each country in our sample. The left–hand panel shows several outliers: China and the EU countries in the sample (Czechia, Hungary, and Poland), while the right–hand panel shows the rest of the countries. We exclude outliers from our regressions and focus only on the sample of countries listed in the right–hand panel.

32

We find statistically significant evidence for the first prediction. Our second test results in a coefficient with the predicted sign, yet statistically insignificant.

In order to test the first hypothesis, we proceed as follows. For each in our sample, we estimated the following time series regression:

$$\pi_t^i = \gamma_0 + \gamma_1 \cdot Ret\_MSCIACWorld_t + \Gamma \cdot Ret\_DomesticStockIndex_t^i + \pi_t^{res,i}, \tag{E.1}$$

where π_t^i is the domestic monthly inflation rate in and $Ret\_MSCIACWorld_t$ is the monthly return on the MSCI AC World Index. $Ret\_DomesticStockIndex_t^i$ is the monthly return on the domestic stock market index. $\pi_t^{res,i}$ are the residuals from this regression. We also run the following regression for the US:

$$\pi_t^{US} = \mu_0 + \mu_1 Ret\_MSCIACWorld_t + \pi_t^{res,US}, \tag{E.2}$$

We then run the following regression to compute a proxy for the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ between the residual domestic inflation and residual US inflation (see item (a) of Proposition E.1),

$$\pi_t^{res,i} = \alpha + \beta \pi_t^{res,US} + \epsilon_t,$$

where $\pi_t^{res,i}$ is the residual domestic monthly inflation rate in from (E.1) and $\pi_t^{res,US}$ is the residual monthly inflation rate in the US from (E.2). We denote the estimated slope coefficient by $\hat{\beta}_i^{\pi_t^{res,i}, \pi_t^{res,US}}$.

We then run the following cross–sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_1 + \beta_1 \hat{\beta}_i^{\pi_t^{res,i}, \pi_t^{res,US}} + X_i + \eta_i. \tag{E.3}$$

Here, $\frac{L\bar{C}U}{USD}_i$ is the average ratio of debt denominated in local currency to debt denominated in dollars for corporates in the countries of the dataset; X_i denotes other control variables.

Item (a) of Proposition E.1 predicts that the coefficient β_1 in the regression (E.3) should be positive.

To test the second hypothesis, we calculate the standard deviation of $\pi_t^{res,i}$ as a proxy for $\sigma_{\varepsilon,i}$

in Proposition E.1, and then run the following cross–sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_2 + \beta_2 \sigma_i^{\pi_t^{res,i}} + X_i + \eta_i \,. \tag{E.4}$$

Proposition E.1, item (b) predicts that $\beta_2 < 0$.

In column (1), we run univariate regressions In column (2), we add an additional control variable $ka\bar{o}pen_i$: a financial openness index obtained from Chinn and Ito (2006). In column (3), we take the predictions of the model literally as they appear in item (a) of the Proposition E.1: $\beta_1 > 0$ for countries where $\frac{L\bar{C}U}{USD}_i > 1$ and we exclude Hong Kong where $\frac{L\bar{C}U}{USD}_i < 1$. In all three columns, regressions corroborate our hypothesis.[3] The first three columns are in line with the predictions of our theory. Column (4) of Table E.1 shows the results of regression (E.4). Although the result is lacking statistical significance, the sign of the coefficient is indeed consistent with our theoretical prediction.

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[3]All our results are qualitatively and quantitatively similar when we use raw domestic and US inflation rates, instead of residuals. Moreover, all results remain valid if we use the share of local currency debt in total debt instead of the ratio of local currency debt to dollar debt.

Table E.1. The cross–section of the local currency to dollar debt ratio

	(1) $\frac{L\bar{C}U}{USD}_i$	(2) $\frac{L\bar{C}U}{USD}_i$	(3) $\frac{L\bar{C}U}{USD}_i$	(4) $\frac{L\bar{C}U}{USD}_i$
$\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$	6.523***	6.094***	6.019***	
	(0.896)	(1.097)	(1.029)	
$ka\bar{o}pen_i$		−0.233	−0.192	−0.796*
		(0.347)	(0.428)	(0.398)
$\sigma_i^{\pi_t^{res,i}}$				−1.784
				(1.479)
Observations	17	17	16	17
R–squared	0.697	0.709	0.664	0.254

Notes: Robust standard errors in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. $\frac{L\bar{C}U}{USD}_i$ is the mean share of local currency debt obtained from the IIF for each of the 17 emerging market economies between 2005 Q1 and 2019 Q4. $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$ is the estimated regression coefficient for a linear regression of residuals of monthly domestic inflation rate from (E.1) on the residuals of the US inflation rate from (E.2). $ka\bar{o}pen_i$ is the mean of the Chinn–Ito financial openness index for each country (average of the data available between 1970–2018). $\sigma_i^{\pi_t^{res,i}}$ is the standard deviation of the residuals of the monthly domestic inflation rate obtained from (E.1). In column (3), Hong Kong is excluded since the share of local currency debt to dollar debt is less than 1.

E.2 Proof of Proposition E.1

We first state the following extension of the Theorem 2.1 for the case of firms borrowing both in local currency and in dollars.

Theorem E.2 *Suppose that $q = q(\$)$. Then, issuing in a mixture of local currency and dollars is optimal if and only if*

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} - 1 \leq \frac{\text{Cov}_t^\$ \left(\left(\frac{\Omega_{t+1}}{\mathcal{B}_{t+1}(B_t)} \right)^{-\ell} , \mathcal{E}_{j,t,t+1} \right)}{E_t^\$ \left[\left(\frac{\Omega_{t+1}}{\mathcal{B}_{t+1}(B_t)} \right)^{-\ell} \right] E_t^\$ [\mathcal{E}_{j,t,t+1}]}$$

for all $j = 1, \cdots, N$.

Proof of Theorem 2.1 and Proposition E.1. The standard Kuhn–Tucker conditions that borrowing only in LC and dollars is optimal if and only if

$$E_t \left[M_{t,t+1} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^\ell \right) (1+c)\mathcal{E}_{j,t+1} \right] \right] (1 - q(j))$$

$$+ E_t \left[M_{t,t+1} \left[\left(-\ell(1-\rho) \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^{\ell-1} \Omega_{t+1}^{-1} \right) (1+c)\mathcal{E}_{j,t+1} \right] \mathcal{B}_{t+1}(B_t) \right] (1 - q(\$))$$

$$- (1 + c(1-\tau)) E_t \left[M_{t,t+1}\mathcal{E}_{j,t+1} \right]$$

$$+ E_t \left[M_{t,t+1}(\ell+1) \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^\ell (1 + c(1-\tau))\mathcal{E}_{j,t+1} \right.$$

$$\left. - \ell \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^\ell (1 + c(1-\tau))\mathcal{E}_{j,t+1} \right] \leq 0$$

for all j with the identity for $j = i, \$$. This inequality can be rewritten as

$$\bar{q}(j,\$) \frac{E_t[M_{t,t+1}\mathcal{E}_{j,t+1}]}{E_t \left[M_{t,t+1} \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^\ell \mathcal{E}_{j,t+1} \right]} \leq 1 = \bar{q}(\$) \frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t \left[M_{t,t+1} \left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}} \right)^\ell \mathcal{E}_{\$,i,t+1} \right]}$$

and the first claim follows.

For the LC–$ mixture, we assume for simplicity that $\ell = 1$. Then, we get the system

$$1 \;=\; \bar{q}(\$) \frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)\mathcal{E}_{\$,i,t+1}\right]}$$

$$1 \;=\; \bar{q}(\$) \frac{E_t[M_{t,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)\right]}$$

whereby

$$\mathcal{B}_{t+1}(B_t) \;=\; (1 + c(1-\tau))(B_t \;+\; B_t(\$)\mathcal{E}_{\$,i,t+1})$$

Thus, we get the system

$$E_t[M_{t,t+1}\Omega_{t+1}^{-1}]B_t \;+\; E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t(\$) \;=\; \tilde{q}(\$)E_t[M_{t,t+1}]$$

$$E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t \;+\; E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]B_t(\$) \;=\; \tilde{q}(\$)E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]$$

where we have defined

$$\tilde{q}(\$) = \bar{q}(\$)/(1 + c(1-\tau)).$$

Thus,

$$\begin{pmatrix} B_t \\ B_t(\$) \end{pmatrix} \;=\; \tilde{q}(\$)\Delta_t^{-1} \begin{pmatrix} E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2] & -E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}] \\ -E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}] & E_t[M_{t,t+1}\Omega_{t+1}^{-1}] \end{pmatrix} \begin{pmatrix} E_t[M_{t,t+1}] \\ E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}] \end{pmatrix}.$$

where

$$\Delta_t \;=\; E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]E_t[M_{t,t+1}\Omega_{t+1}^{-1}] - (E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}])^2$$

Thus,

$$\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \;=\; \frac{-\mathrm{Cov}_t^{\$}(\Omega_{t+1}^{-1}\mathcal{E}_{t,t+1,\$,i}, \; \mathcal{E}_{t,t+1,\$,i}^{-1})}{\mathrm{Cov}_t^{\$}(\Omega_{t+1}^{-1}, \; \mathcal{E}_{t,t+1,\$,i}^{-1})}.$$

Thus,

$$\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \;=\; \frac{-\mathrm{Cov}_t^{\$}\left((\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1})^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}{\mathrm{Cov}_t^{\$}\left((\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1})^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}.$$

Let now $\tilde{a}_{i,t+1} \equiv \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}) - \beta\tilde{a}_{\$,t+1}$ where $\tilde{a}_{\$,t+1} = \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{\$,t+1}})$ and where β is such that $\tilde{a}_{i,t+1}$ and $\tilde{a}_{\$,t+1}$ are uncorrelated.

Recall also that we assume that

$$\log\mathcal{P}_{i,t,t+1} = -\hat{\alpha}_i\tilde{a}_{\$,t+1} - \alpha_i\tilde{a}_{i,t+1} + \varepsilon_{i,t+1}, \ \log\mathcal{P}_{\$,t,t+1} = -\tilde{\alpha}_\$\hat{a}_{\$,t+1} + \varepsilon_{\$,t+1}$$

where $\varepsilon_{i,t+1} \sim N(0,\sigma_{\varepsilon,i}^2)$. We also allow $\sigma_{\varepsilon,i,\$} \equiv \text{Cov}_t(\varepsilon_{i,t+1},\varepsilon_{\$,t+1}) \neq 0$. Then, to the first order in variance, the measure change is irrelevant and

$$- \text{Cov}_t^\$ \left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)$$

$$\approx -\text{Cov}_t(-\tilde{a}_{i,t+1} - \beta\tilde{a}_{\$,t+1} + \alpha_\$\tilde{a}_{\$,t+1} - \varepsilon_{\$,t+1}, -\alpha_\$\tilde{a}_{\$,t+1} + \varepsilon_{\$,t+1} + \alpha_i\tilde{a}_{i,t+1} + \hat{\alpha}_i\tilde{a}_{\$,t+1} - \varepsilon_{i,t+1})$$

whereas

$$\text{Cov}_t^\$ \left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)$$

$$\approx \text{Cov}_t(-\tilde{a}_{i,t+1} - \beta\tilde{a}_{\$,t+1} + \alpha_i\tilde{a}_{i,t+1} + \hat{\alpha}_i\tilde{a}_{\$,t+1} - \varepsilon_{i,t+1},$$

$$- \alpha_\$\tilde{a}_{\$,t+1} + \varepsilon_{\$,t+1} + \alpha_i\tilde{a}_{i,t+1} + \hat{\alpha}_i\tilde{a}_{\$,t+1} - \varepsilon_{i,t+1})$$

In the small variance approximation, we that's get

$$\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \approx \frac{\sigma_{\varepsilon,\$}^2 - \sigma_{\varepsilon,i,\$} + \alpha_i\sigma_c^2 + \alpha_\$^2\sigma_c^2(\$) - (\alpha_\$ + \alpha_i\alpha_\$)\sigma_c(i,\$)}{\sigma_{\varepsilon,i}^2 - \sigma_{\varepsilon,i,\$} + (1-\alpha_i)(\alpha_\$\sigma_c(i,\$) - \alpha_i\sigma_c^2)}$$

where $\sigma_c^2 = \text{Var}_t[\log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}})]$ and $\sigma_c(i,\$) = \text{Cov}_t[\log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}), \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{\$,t+1}})]$.

The claims (monotonicity in $\sigma_{\varepsilon,i,\$}$ and $\sigma_{\varepsilon,i}^2$) follow then by direct calculation. Q.E.D.

References

Bodnár, K., 2006. Survey evidence on the exchange rate exposure of hungarian smes. MNB Bulletin 1, 6–12.

Bruno, V., Shin, H. S., 2017. Global dollar credit and carry trades: a firm-level analysis. Review of Financial Studies 30, 703–749.

Chinn, M. D., Ito, H., 2006. What matters for financial development? capital controls, institutions and interactions. Journal of Development Economics 81, 163–192.

Du, W., Schreger, J., 2017. Sovereign risk, currency risk and corporate balance sheets. Tech. rep., Federal Reserve Board.

Froot, K. A., Scharfstein, D. S., Stein, J. C., 1993. Risk management: Coordinating corporate investment and financing policies. The Journal of Finance 48, 1629–1658.

Hördahl, P., Tristani, O., 2014. Inflation risk premia in the euro area and the united states. International Journal of Central Banking 10, 1–47.

Hugonnier, J., Malamud, S., Morellec, E., 2015. Capital supply uncertainty, cash holdings and investment,. Review of Financial Studies 28, 391–445.

Kremens, L., Martin, I. W., 2019. The quanto theory of exchange rates. American Economic Review 109, 810–843.

Niepmann, F., Schmidt-Eisenlohr, T., 2019. Foreign currency loans and credit risk: Evidence from u.s. banks. Tech. rep., Federal Reserve Board.

Rampini, A., Sufi, A., Viswanathan, S., 2014. Dynamic risk management. Journal of Financial Economics 111, 271–296.

Salomao, J., Varela, L., forthcoming. Exchange rate exposure and firm dynamics. Review of Economic Studies .